MATHEMATICS

DPP

DAILY PRACTICE PROBLEMS

DPP No. 32

Total Marks : 29

Max. Time : 31 min.

Topics : **Method of Differentiation, Complex Number, Continuity & Derivability, Application of Derivatives, Sequence & Series, Function**

Type of Questions		M.M., Min.
Single choice Objective (no negative marking) Q.1,2,3,4	(3 marks, 3 min.)	[12, 12]
Multiple choice objective (no negative marking) Q.5	(5 marks, 4 min.)	[5, 4]
Subjective Questions (no negative marking) Q.6,7,8	(4 marks, 5 min.)	[12, 15]

1. Let $y = \tan^{-1}\left(\frac{2\cos(3x^2-2)+5\sin(3x^2-2)}{5\cos(3x^2-2)-2\sin(3x^2-2)}\right)$, then $\frac{dy}{dx}$ =

(A) $6x - 2$ (B) $6x$ (C) $5x$ (D) $\frac{6x}{x^2+1}$

2. If $y = at^2 + 2bt + c$ and $t = ax^2 + 2bx + c$, then $\frac{d^3y}{dx^3}$ equals

(A) $24a^2\,(at + b)$ (B) $24a\,(ax + b)^2$ (C) $24a\,(at + b)^2$ (D) $24a^2\,(ax + b)$

3. The complex number $z = x + iy$ for which $\log_{1/2}|z-2| > \log_{1/2}|z|$, are given by:

(A) Re (z) ≤ 1 (B) Im (z) ≤ 1 (C) Re (z) > 1 (D) Im (z) > 1

4. If $g(x) = \frac{2h(x)+|h(x)|}{2h(x)-|h(x)|}$ where $h(x) = \sin x - \sin^n x$, $n \in R^+$, the set of positive real numbers, and

$$f(x) = \begin{cases} [g(x)], & x \in \left(0, \frac{\pi}{2}\right)\cup\left(\frac{\pi}{2}, \pi\right) \\ 3, & x = \frac{\pi}{2} \end{cases}$$

where [.] denotes greatest integer function. Then

(A) f(x) is continuous and differentiable at $x = \frac{\pi}{2}$, when $0 < n < 1$

(B) f(x) is continuous and differentiable at $x = \frac{\pi}{2}$, when $n > 1$

(C) f(x) is continuous but not differentiable at $x = \frac{\pi}{2}$, when $0 < n < 1$

(D) f(x) is continuous but not differentiable at $x = \frac{\pi}{2}$, when $n > 1$

5. For the series $S = 1 + \frac{1}{(1+3)}(1+2)^2 + \frac{1}{(1+3+5)}(1+2+3)^2 + \frac{1}{(1+3+5+7)}(1+2+3+4)^2 + \ldots..$

(A) 7^{th} term is 16 (B) 7^{th} term is 18

(C) sum of first 10^{th} terms is $\frac{505}{4}$ (D) sum of first 10^{th} term is $\frac{405}{4}$

6. Let $f(x) = \frac{1}{1-x}$, g(x) = fofofofofofof(x) and $h(x) = \tan^{-1}(g(-x^2-x))$, then find $\lim_{n\to\infty}\sum_{r=1}^{n} h(r)$.

7. Prove that in the curve $y = a\,\ell n\,(x^2 - a^2)$, sum of the tangent and subtangent varies as the product of the coordinates of the point of contact.

8. If the equation $a_0 x^n + a_1 x^{n-1} + \text{-------} + a_{n-1} x = 0$, (where n is natural number) has a positive root α, prove that the equation $na_0 x^{n-1} + (n-1)a_1 x^{n-2} + \text{-------} + a_{n-1} = 0$ also has a positive root smaller than α.

Answers Key

1. (B) **2.** (D) **3.** (C) **4.** (B)

5. (A)(C) **6.** $\frac{\pi}{4}$